Multipodal phases in graphs
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The first third of the talk is devoted to setting up the thermodynamic formalism for large, dense, random graphs. What is a random graph? What are graphons? What are the different notions of entropy, and why are they the same?

In the middle third, I'll present the edge-triangle model and explore the different phases and different sorts of phase transitions that occur. It is observed that all phases are "multipodal", meaning that the vertices of typical graphs group into a finite number of clusters, with edges between nodes in cluster $i$ and nodes in cluster $j$ occurring independently with probability $p_{ij}$. This is backed up by extensive numerics and perturbative expansions, but has only been proven rigorously along some special curves.

In the last third I'll present a different model, involving "k-stars" instead of triangles, and prove that all phases are multipodal. Finally, I'll present some conjectures and open problems.